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FREQUENCY PARAMETERIZATION OF OPEN STRUCTURES BY MODAL DECOMPOSITION USING FINITE ELEMENTS AND SPHERICAL MODES

Y. Adane**, A. Gati**, M. F. Wong**, Senior IEEE, V. Fouad Hanna*, Fellow IEEE
* Laboratoire Instruments et Systèmes - Case 252 - Université Pierre et Marie Curie
4, place Jussieu - 75252 Paris cedex 05
** France Télécom R&D - DMR/IIM
38-40, Rue du Général Leclerc - 92794 Issy Moulineaux cedex 09

Abstract— **This paper presents the extension of a modal decomposition technique for the frequency parameterization of open structures. The modal decomposition based on the computation of the poles and modes of 3D geometries allows to get a broadband characterization of a microwave structure. So far, the technique is restricted to closed structures. With the help of spherical modes for the outer region of open structures, full-wave 3D models parameterized in frequency can be obtained. These models can be derived efficiently from few rigorous computations at given frequencies using finite-element method.**

I. Introduction

For the purpose of analysis and design of open or closed microwave circuits, we need to determine their frequency performance in a broadband in which the response is not necessarily smooth. For this purpose when applied on complex structures, full-wave 3D electromagnetic modelling is required. Time domain methods like the Finite Difference Time Domain can provide broad band frequency responses. Alternatively, frequency domain methods like the Finite Element Method (FEM) enhanced by techniques so-called fast frequency sweep can also derive efficiently broad band frequency responses avoiding missing the abrupt forced frequency variations when using point by point sampling. Such a technique has been successfully developed based on a modal field expansion [1, 2]. This technique is particularly competitive for analysing structures like filters which require an important number of time steps, when using time domain methods, to determine long transients or an important number of frequency steps, when using classical frequency domain methods to detect sharp resonances. Furthermore, this technique enables the full-wave 3D modeling of microwave circuits in both frequency and geometry which opens the way for electromagnetic based sensitivity analysis and optimisation.

In this work, a modal decomposition technique is presented for the frequency parametrization of open structures. The electromagnetic field analysis of the open structure is performed using the FEM coupled to the spherical modes. A general procedure can be followed to get the desired modal expansion with respect to the frequency provided solving the problem at two frequencies. The result is a 3D full-wave model of a radiating structure parameterized in frequency.

II. Theory

Let an open structure sketched in its electrical equivalent representation in Fig. 1. The structure comprises the inner region or core region and the outer free space region. The core region will be analyzed using FEM while the free space domain will be described using spherical modes [3]. Thus, the boundary surface is a sphere.

Let's consider a transmitting open structure placed in a spherical coordinate system, we denote r_0 as the radius of the smallest possible spherical surface with the

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center at the origin chosen at the interior of the radiating structure. The space outside the minimum sphere can be viewed as a space in which the propagation is done in the radial direction. And the "waveguide" modes are TE and TM spherical modes.

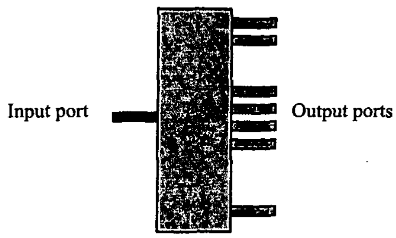

Fig. 1 . Equivalent representation of an open structure in the open space

The general antenna with an associated spherical coordinate system may be considered as a waveguide junction with several ports. The port connected to the generator is the input port. The remaining ports are the output ports that are connected to equivalent modal transmission lines, one for each spherical mode

For the junction region, i.e. the inner region, we use the FEM [4]. This technique leads to the linear equation [4]:

$$\left(R - k^2 M\right) e = jk\eta \sum_n J_{epn} \qquad (1)$$

Where R and M are the rigidity and the mass matrices respectively and J_{epn} the excitation array containing the excitation input courant and the spherical modes of the output ports.

Recently, a modal decomposition based method [1] has been proposed to solve the equation (1). The method outputs a frequency and geometry-parameterized model which allows sensitivity analysis and optimization [1]. In this method, the E field propagating inside the cavity can be expressed as a decomposition on the cavity modes. Hence, for each excited port noted by n we obtain the solution as :

$$e_n = j\eta\, k \sum_i \frac{C_{ni}}{k_i^2 - k^2} V_i \qquad (2)$$

Where k_i are the eigenvalues and the V_i eigenvectors of the system (1). C_{ni} is the scalar product between J_{epn} and V_i which corresponds to the coupling between the waveguide modes and the cavity modes.

Using (2), the impedance matrix [Z] can be calculated, since each impedance term associated to the ports noted n and m, is given by the relation ($Z_{nm} = {}^{t}J_{epn}\, e_m$) [4]. The pole expansion of the impedance matrix is therefore written in the form [1]:

$$Z_{nm} = j\eta\, k \sum_i \frac{C_{ni} C_{mi}}{k_i^2 - k^2} \qquad (3)$$

III. Modal expansion for the radiating structures

The analytical formulas (2) and (3) give the broadband frequency response of the microwave structure. These can be viewed as the analytical transfer functions in terms of a pole expansion.

The relation (3) can be further decomposed as [1] :

$$Z_{mn} = \frac{jA_{mn}}{k} + jkB_{mn} + jk^3\eta \sum_i^{Q} \frac{C_{ni}.C_{mi}}{k_i^2\left(k_i^2 - k^2\right)} \qquad (4)$$

$$\text{Where: } \begin{cases} A_{nm} = \sum_{i=1}^{Q} C_{ni} C_{mi} \\ B_{nm} = \sum_{i=1}^{Q} \frac{C_{ni} C_{mi}}{k_i^2} \end{cases} \qquad (5)$$

The relation (4) highlights the frequency independent A_{nm} and B_{nm} terms. A_{nm} represents the eventual contribution of the poles at the zero frequency and may be determined by using asymptotic approximation. B_{nm} converges slowly and thus requires a lot of eigenvalues to be determined. Hence, it's better to determine it by an asymptotic approach too. In the case of closed structures, assumptions are made in order to get low frequency asymptotic asymptotic expressions to match with the constants A_{nm} and B_{nm}.

To illustrate this and as a test, we consider an homogeneous free space sphere. The impedance matrix is known analytically. The truncated series expansion of the spherical TE and TM modes impedance are :

$$Z_{TEmn} = -j\left(\frac{kr}{n+1} + o(k^3r^3)\right) \quad (5)$$

$$Z_{TMmn} = j\left(\frac{n+1}{kr} - \frac{kr}{2n+3} + o(k^3r^3)\right) \quad (6)$$

Where r is the radius of the sphere.

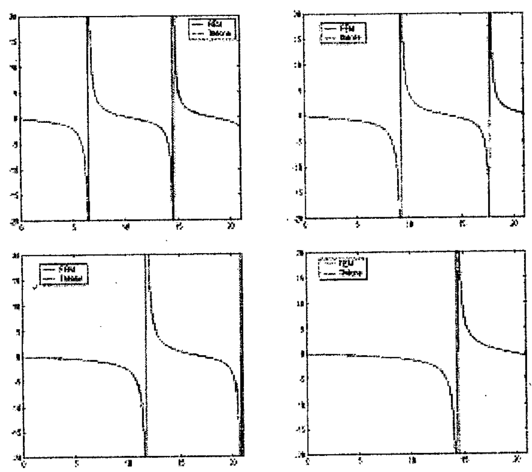

Fig. 2 . Impedance of the TE_{m1}, TE_{m2}, TE_{m3}, TE_{m4}, spherical modes

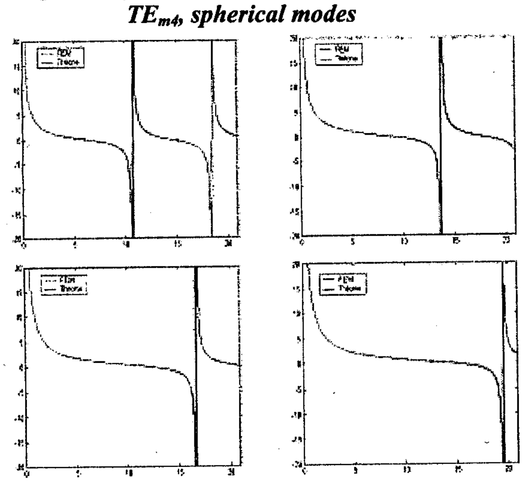

Fig. 3 . Impedance of the TM_{m1}, TM_{m2}, TM_{m3}, TM_{m4}, spherical modes

Figure 2 (resp. 3) shows in red the impedance of some TE (resp. TM) spherical modes calculated with the relation (5) (resp. 6) in the frequency band going from 0 to 20 Ghz. These curves are compared to the theoretical curves (in blue). We notice that the curves are coincident.

Although these results are encouraging, it is not possible to use this method to characterize an open structure surrounded by a sphere, since we do not have such available formulas (5) and (6) in the general case. To overcome this problem, the proposed method is as follows. First, we calculate the impedance matrix of the studied structure for two different frequencies. Second, we calculate the C_{nm} coefficients characterizing the same structure. Finally, with these known parameters we can deduce the terms A_{nm} and B_{nm}.

IV. Results

As a test case, we consider an Hertz dipole. After determining the terms A_{nm} and B_{nm}, then using the equation (6), we find the frequency-parameterized impedance matrix and calculate the scattering matrix of the dipole. Moreover, we have established the radiation patterns for many frequencies (fig 4). These patterns are compared with the ones calculated by a direct frequency solving using FEM. We notice that figures 4 and 5 represent the well-known radiation pattern of a dipole, and the two figures agree very well.

V. Conclusion

The modal decomposition method based on the poles and modes calculation of a microwave structure has been used for the frequency parametrization of open structures. A simple procedure has been proposed that enables the derivation of fullwave 3D models parameterized in frequency.

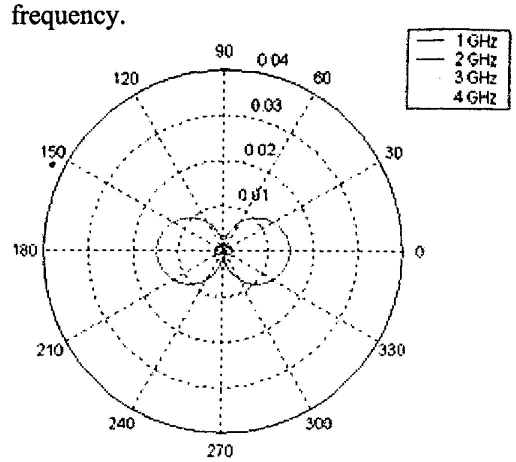

Fig. 4 . Representation of the electric field of the dipole obtained with a courant of 1 A, calculated by modal decomposition

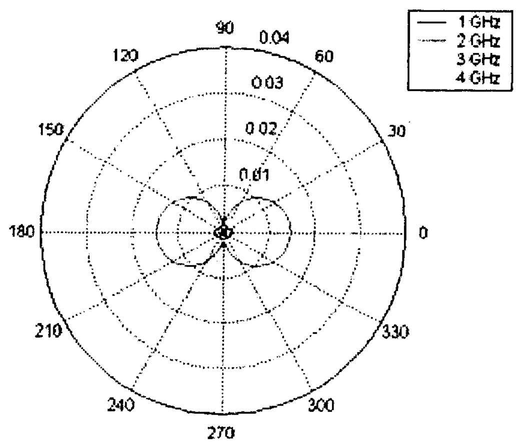

Fig. 5 . Representation of the electric field of the dipole obtained with a courant of 1 A, calculated by linear resolution

VI. References

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